Balancing

Balancing is the process of eliminating or at least reducing the ground forces and/or moments. It is achieved by changing the location of the mass centers of links. Balancing of rotating parts is a well known problem. A rotating body with fixed rotation axis can be fully balanced i.e. all the inertia forces and moments. For mechanism containing links rotating about axis which are not fixed, force balancing is possible, moment balancing by itself may be possible, but both not possible. We generally try to do force balancing. A fully force balance is possible, but any action in force balancing severe the moment balancing.

Rotating Mass Balance, Static and Dynamic Balance

A geometrically symmetrical body is expected to be balanced. Due to non-uniform distribution of material like air voids due to bad casting, mass center may not be coinciding with the fixed pivot. So, every rotating component must be checked for unbalance. Fixed axis rotation of a body mass center is not coinciding with rotation axis O, shown in figure. Centripedal force $F$ is $m\omega^2 r$.

![Diagram of fixed axis rotation of a body](image)

Fig. Fixed axis rotation of a body. Mass center is not coinciding with rotation axis O.

We want, $m\omega^2 r = 0$, possible cases;
$m = 0 :$ mathematically correct but physically trivial,
$\omega = 0 :$ mathematically correct but physically trivial,
$r = 0 :$ mathematically correct yet physically realisable by balancing.

How can we understand if a body has any unbalance?
A simple experiment may be conducted as;
We place the rotating body on frictionless bearings or knife edge bars as depicted in figure 2, and by turning it a bit and then waiting until it comes to rest. So, bottommost point is marked. That mark represents the heavier side of the body. We disturb the body from its rest position by turning it again a bit and letting it comes to rest. If the mark is still at the bottommost position, then this side surely heavier than the opposing side. We either remove material from the heavier side or add material to the lighter side for correction. Then, repeat the procedure until no part of the body comes insistively to the bottommost position.

Figure 2, Simple experiment

In this process we make use of the gravity force only which is \( mg \). Both \( m \) and \( g \) are constant so, the force which we make use of in balancing is a “static” force. And the method is called “static balancing”. In such balancing, we can not decide or see inspect the amount of unbalance and orientation. Unbalance is found and its correction is done by an experimental approach using a balancing machine.

Static balancing is suitable for rotating objects with negligible axial dimensions like fans, thin flywheels etc. Static balancing of these components provide both force and moment balance. If axial dimensions are not negligible a simple static balance will not ensure moment balance though it ensures force balance. To display the moment unbalance, body must be turning and noncollinear centrepedal forces must be generated, so forces we make use of are \( m\omega^2r \). Body is no longer static hence balancing done in this format called the “dynamic balancing”.

Dynamic balancing is done by adding or removing mass at two places known as the “correction planes”. Distance between the correction planes must be as much as possible and
the designer must facilitate large and heavy portions at each end of a shaft where removing some material by drilling does not reduce the strength of the shaft.

Analysis of Unbalance
In this section we will learn how to analyze any unbalanced rotating body and determine the proper correction. Firstly, we will discuss the graphical method, secondly, vector methods.

Graphical Method
When the rotating masses are in different planes as shown in figure, following two equations should be satisfied;

\[ \sum F = 0 \quad \text{and} \quad \sum M = 0 \]

Consider the shaft depicted in figure is to be balanced. We need to determine the location and amount of correction masses. We begin by moment balancing. Summation of the centripetal forces, including the corrections about some point. We choose LCP point. Thus applying the

\[ \sum \ddot{M} = 0 \] gives

\[ \sum M_{LCP} = 0; \quad \ddot{M}_1 + \ddot{M}_2 + \ddot{M}_{RCP} = 0 \]

Moment generated by the first unbalance \( M_1 \) is \( l_1 m_1 r_1 \omega^2 \) with respect to LCP.

Moment generated by the second unbalance \( M_2 \) is \( l_2 m_2 r_2 \omega^2 \) with respect to LCP.
As $\omega$ is common in all vector for simplicity we can drop it. Taking the directions of the moments as if forces directions. A moment polygon, depicted in figure, can be constructed. Note that a true moment directions would be obtained by rotating the polygon 90 degree CW.

Magnitude and direction of the correction factor is determined from the moment polygon. Its magnitude is $m_{rep}r_{rep}l$. We know $l$ by design. By dividing $l$, we get $m_{rep}r_{rep}$. Any combination of $m_{rep}$ and $r_{rep}$ providing arithmetical conforming will do. You can put a small mass at a long $r_{rep}$ or large mass at a small $r_{rep}$. With the addition of $m_{rep}$, moment balance is done. But, $\sum \vec{F} = 0$ equation is not satisfied yet. For force balance, we do not need to rotate the shaft, we can make use of gravity forces only. Similiarly, magnitude and direction of the correction mass to be placed LCP can be found from force polygon.
**Example 1**

a) In the figure an unbalanced shaft with non-negligible axial dimensions is shown. It is to be balanced by putting balance masses at the correction planes. Calculate the amount of masses to be used at a radial distance of 0.3 m on the correction planes.

\[ \sum F = 0; \quad F_{LCP} + F_{RCP} + F_1 + F_2 = 0 \]

where
\[ F_1 = m_1r_1 = 4 \times 0.1 = 0.4 \text{kgm} \uparrow \]
\[ F_2 = m_2r_2 = 2 \times 0.2 = 0.4 \text{kgm} \uparrow \]
\[ F_{LCP} = m_{LCP}r_{LCP} = ? \]
\[ F_{RCP} = m_{RCP}r_{RCP} = ? \]

\[ m_{LCP} = \frac{F_{LCP}}{r_{LCP}} = \frac{0.4}{0.3} = 1.333 \text{kg} \downarrow \text{ANS} \]

\[ F_{RD} = 0.4 \text{ kgm} \]
\[ F_2 = 0.4 \text{ kgm} \]
\[ F_{LD} = 0.4 \text{ kgm} \]
\[ F_1 = 0.4 \text{ kgm} \]

\[ F_{RCP} = 0.3 \text{ kgm} \]
\[ F_{RCP} = 0.4 \text{ kgm} \]

\[ m_{RCP} = \frac{F_{RCP}}{r_{RCP}} = \frac{0.4}{0.3} = 1.333 \text{kg} \downarrow \text{ANS} \]

\[ \sum M_{LCP} = 0; \quad M_{RCP} + M_1 + M_2 = 0 \]

where
\[ M_1 = I_1 F_1 = 0.25 \times 0.4 = 0.1 \text{kgm}^2 \uparrow \]
\[ M_2 = I_2 F_2 = 0.5 \times 0.4 = 0.2 \text{kgm}^2 \uparrow \]
\[ M_{RCP} = l_{RCP} F_{RCP} = ? \]
\[ M_{RCP} = 0.3 \text{kgm}^2 = l_{RCP} F_{RCP} = 0.75 \times F_{RCP} \]
\[ F_{RCP} = 0.3 \div 0.75 = 0.4 \text{kgm} \]
\[ m_{RCP} = \frac{F_{RCP}}{r_{RCP}} = \frac{0.4}{0.3} = 1.333 \text{kg} \downarrow \text{ANS} \]

\[ M_{RCP} = 0.3 \text{kgm}^2 \]
\[ M_2 = 0.2 \text{kgm}^2 \]
\[ M_1 = 0.1 \text{kgm}^2 \]
b) Can this shaft be dynamically balanced by putting a single mass? If so, where must be the correction plane with radial distances 0.3 m.

YES. Because all masses (including correction masses) are on the single plane.

\[ \sum F = 0; \quad F_c = F_{RCP} + F_{LCP} = 0.8kgm = m_c r_c \Rightarrow m_c = \frac{0.8}{0.3} = 2.66kg \quad \text{ANS} \]

\[ \sum M_{LCP} = 0; \quad M_c = x \cdot F_c \Rightarrow 0.3 = x \cdot 0.8 \Rightarrow x = \frac{0.3}{0.8} = 0.375m \quad \text{From LCP} \quad \text{ANS} \]

**Example** In the figure a rotating shaft is shown. The shaft rotates at 750 rpm. It is supported in bearings at A and B. 

\( m_1 = 2 \text{ kg}, \quad m_2 = 2 \text{ kg}, \quad r_1 = 0.4m, \quad r_2 = 0.2m \)

a) What are the bearing reactions for the systems?

\[ \omega = 750\text{ rev/min} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{2\pi}{1 \text{ rev}} = 78.54\text{ rad}. \]

\[ F_1 = m_1 r_1 \omega^2 = 2 \times 0.4 \times 78.54^2 = 4934.83N \uparrow \]

\[ F_2 = m_2 r_2 \omega^2 = 2 \times 0.2 \times 78.54^2 = 2467.41N \downarrow \]

\[ F_A = \ ? \quad \& \quad F_B = \ ? \]

Taking moment about A,

\[ \sum M_A = 0; \quad 0.3 \cdot F_1 - 0.3 \cdot F_2 - 0.6 \cdot F_B = 0 \Rightarrow F_B = \frac{0.3 \cdot 4934.83 - 0.3 \cdot 2467.41}{0.6} = 1233.71N \downarrow \]

\[ \sum F_y = 0; \quad -F_A - F_B - F_2 + F_1 = 0 \Rightarrow F_A = 4934.83 - 2467.41 - 1233.71 = 1233.71N \downarrow \text{ANS} \]
b) Determine the location and magnitude of a balancing mass if it is to be placed at a radius of 0.3 m.

\[ \sum \vec{F} = 0; \quad \vec{F}_c + \vec{F}_1 + \vec{F}_2 = 0 \]

where

\[ F_1 = m_1 r_1 \omega^2 = 2 \times 0.4 \times 78.54^2 = 4934.83 \text{N} \uparrow \]
\[ F_2 = m_2 r_2 \omega^2 = 2 \times 0.2 \times 78.54^2 = 2467.41 \text{N} \downarrow \]
\[ F_c = m_3 r_3 \omega^2 = 4934.83 - 2467.41 = 2467.41 \text{N} \downarrow \]

\[ m_c = \frac{F_c}{r_c \omega^2} = \frac{2467.41}{0.3 \times 78.54^2} = 1.333 \text{kg} \downarrow \text{ANS} \]

c) Calculate the bearing reactions after adding the balancing mass. Compare and discuss your results before and after balancing.

\[ F_1 = m_1 r_1 \omega^2 = 2 \times 0.4 \times 78.54^2 = 4934.83 \text{N} \uparrow \]
\[ F_2 = m_2 r_2 \omega^2 = 2 \times 0.2 \times 78.54^2 = 2467.41 \text{N} \downarrow \]
\[ F_c = m_3 r_3 \omega^2 = 4934.83 - 2467.41 = 2467.41 \text{N} \downarrow \]
\[ F_A = ? \quad \text{&} \quad F_B = ? \]

Taking moment about A,

\[ \sum M_A = 0; \quad 0.3 \times F_1 - 0.3 \times F_2 - 0.3 \times F_c - 0.6 F_B = 0 \Rightarrow F_B = 0 \text{ then } F_A = 0 \text{ ANS.} \]
Example: The shaft shown in the figure is rotating with a speed of 10 rad/sec. Calculate the amount and angular orientation of the balance masses to be placed at a radial distance of 0.2 m.

\[ m_1 = 2 \text{ kg}, \quad m_2 = 2.5 \text{ kg}, \quad m_3 = 2 \text{ kg}, \quad r_1 = 0.5 \text{ m}, \quad r_2 = 0.2 \text{ m}, \quad r_3 = 0.2 \text{ m} \]

\[ \sum \mathbf{F} = 0; \quad \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_{RCP} + \mathbf{F}_{LCP} = 0 \]

\[ F_1 = m_1 r_1 \omega^2 = 2 \times 0.5 \times 10^2 = 100 \text{ N} \angle 150^\circ \]

\[ F_2 = m_2 r_2 \omega^2 = 2 \times 0.2 \times 10^2 = 50 \text{ N} \angle 30^\circ \]

\[ F_3 = m_3 r_1 \omega^2 = 2 \times 0.2 \times 10^2 = 40 \text{ N} \angle 270^\circ \]

\[ F_{LCP} = ? \quad F_{RCP} = ? \]

\[ \sum \mathbf{M} = 0; \quad \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_{RCP} = 0 \]

\[ M_1 = F_1 l_1 = 100 \times 0.4 = 40 \text{ N} \angle 150^\circ \]

\[ M_2 = F_2 l_2 = 50 \times 0.8 = 40 \text{ N} \angle 30^\circ \]

\[ M_3 = F_3 l_3 = 40 \times 1 = 40 \text{ N} \angle 270^\circ \]

\[ M_{RCP} = ? \]

\[ M_{RCP} = 0 \]

\[ F_{LCP} = 56 = m_{LCP} \times r_{LCP} \times \omega^2 \Rightarrow m_{LCP} = \frac{56}{100 \times 0.2} = 2.8 \text{ kg} \angle 321^\circ \]
**Example** Figure represents a rotating system that has been idealised for illustrative purposes. A weightless shaft is supported in bearings at A and B rotates at 955 rpm.

\[ m_1 = m_2 = 0.5 \text{ kg}, \quad m_3 = 1 \text{ kg}, \quad r_1 = r_2 = r_3 = 0.2 \text{ m} \]

![Diagram of rotating system](image_url)

(a) What are the bearing reactions for the system?

\[
\omega = n = \frac{955 \text{ rev}}{\text{min}} \times \frac{2\pi}{\text{rev}} \times \frac{1\text{ min}}{60\text{ sec}} = 100 \text{ rad/sec}
\]

\[ \sum \vec{F} = 0; \quad \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_A + \vec{F}_B = 0 \]

\[
F_1 = m_1 r_1 \omega^2 = 0.5 \times 0.2 \times 100^2 = 1000 \text{ N} \angle 150^\circ \\
F_2 = m_2 r_2 \omega^2 = 0.5 \times 0.2 \times 100^2 = 1000 \text{ N} \angle 30^\circ \\
F_3 = m_3 r_3 \omega^2 = 1 \times 0.2 \times 100^2 = 2000 \text{ N} \angle 270^\circ \\
F_A = ? \\
F_B = ?
\]

\[ F_A = 400 \text{ N} \angle 270^\circ \]

\[ \sum M = 0; \quad \sum M_1 + M_2 + M_3 + M_B = 0 \]

\[
M_1 = F_1 l_1 = 1000 \times 1.4 = 1400 \text{ Nm} \angle 150^\circ \\
M_2 = F_2 l_2 = 1000 \times 1.4 = 1400 \text{ Nm} \angle 30^\circ \\
M_3 = F_3 l_3 = 2000 \times 1.4 = 2800 \text{ Nm} \angle 270^\circ \\
M_B = F_B l = ?
\]

\[ F_B = \frac{M_B}{l} = \frac{1400}{1} = 1400 \text{ N} \angle 90^\circ \]
b) Determine the location and magnitude of a balancing mass if it is to be placed at a radius of 0.2 m.

From the moment polygon, moment created by correction mass which is to be placed at the correction plane is equal to 1400 N. Then,

\[ M_{\text{corr}} = F_{\text{corr}} \cdot l = m_{\text{corr}} \cdot r_{\text{corr}} \cdot \omega^2 \cdot l \Rightarrow m_{\text{corr}} = \frac{1400}{0.2 \cdot 100^2 \cdot 1.4} = 0.5 \text{ kg} \]
Example The shaft shown in the figure is rotating with a constant speed of 10 rad/sec. Calculate the amount and angular orientation of the balance masses to be placed at a radial distance of 0.3 m. $m_1 = 2 \text{ kg}$, $m_2 = 4 \text{ kg}$, $m_3 = 3 \text{ kg}$, $r_1 = r_2 = r_3 = 0.2 \text{ m}$.

$$\sum M_{LCP} = 0; M_1 + M_2 + M_3 + M_{RCP} = 0$$

$M_1 = F_1t_1 = 0.5 \times 40 = 20 \text{ Nm} \angle 90^\circ$

$M_2 = F_2t_2 = 80 \times 0.75 = 60 \text{ Nm} \angle 135^\circ$

$M_3 = F_3t_3 = 60 \times 1.25 = 75 \text{ Nm} \angle 315^\circ$

$M_{RCP} = F_{RCP}l = ?$

$M_{RCP} = 76 \text{ Nm} \Rightarrow$

$F_{RCP} = \frac{M_{RCP}}{l} = \frac{76}{1.75} = 43.43 \text{ N} \angle 98^\circ$

$m_{RCP} = \frac{F_{RCP}}{\omega^2r} = \frac{43.43}{10^2 \times 0.3} = 1.448 \text{ kg} \angle 98^\circ$
\[ \sum \vec{F} = 0; \quad \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_r + \vec{F}_b = 0 \]

\[ F_1 = m_1 r_1 \omega^2 = 2 \times 0.2 \times 10^2 = 40 N \angle 90^\circ \]
\[ F_2 = m_2 r_2 \omega^2 = 4 \times 0.2 \times 10^2 = 80 N \angle 135^\circ \]
\[ F_3 = m_3 r_3 \omega^2 = 3 \times 0.2 \times 10^2 = 60 N \angle 315^\circ \]
\[ F_{RCP} = 43.43 N \angle 98^\circ \]
\[ F_{LCP} = ? \]

\[ F_{LCP} = 25.8 N \angle 38.4^\circ \]

\[ m_{LCP} = \frac{F_{LCP}}{\omega^2 r} = \frac{25.8}{10^2 \times 0.3} = 0.86 kg \angle 38.4^\circ \]

**Example** The shaft shown in the figure is rotating with a constant speed of 1 rad/sec. Calculate the amount and angular orientation of the balance masses to be placed at a radial distance of 0.3 m. \( m_1 = 2 \text{ kg}, m_2 = 4 \text{ kg}, r_1 = r_2 = 0.2 \text{ m} \).
\[ \sum M_{LCP} = 0; \quad \bar{M}_1 + \bar{M}_2 + \bar{M}_{RCP} = 0 \]

\[ M_1 = F_1l_1 = 0.5 \times 0.4 = 0.20 \text{ Nm} \angle 90^\circ \]

\[ M_2 = F_2l_2 = 0.8 \times 0.75 = 0.60 \text{ Nm} \angle 135^\circ \]

\[ M_{RCP} = F_{RCP}l = ? \]

\[ M_{RCP} = 0.48 \text{ Nm} \implies \]

\[ F_{RCP} = \frac{M_{RCP}}{l} = \frac{0.48}{1.5} = 0.32 \text{ N} \angle 27.6^\circ \]

\[ m_{RCP} = \frac{F_{RCP}}{\omega^2 r} = \frac{0.32}{1^2 0.3} = 1.067 \text{ kg} \angle 27.6^\circ \]

\[ \sum \vec{F} = 0; \quad \vec{F}_1 + \vec{F}_2 + \vec{F}_{RCP} + \vec{F}_{LCP} = 0 \]

\[ F_1 = m_1r_1\omega^2 = 2 \times 0.2 \times 1^2 = 0.40 \text{ N} \angle 90^\circ \]

\[ F_2 = m_2r_2\omega^2 = 4 \times 0.2 \times 1^2 = 0.80 \text{ N} \angle 135^\circ \]

\[ F_{RCP} = 0.32 \text{ N} \angle 27.6^\circ \]

\[ F_{LCP} = ? \]

\[ F_{LCP} = 0.283 \text{ N} \angle 3.4^\circ \]

\[ m_{LCP} = \frac{F_{LCP}}{\omega^2 r} = \frac{0.283}{1^2 0.3} = 0.943 \text{ kg} \angle 3.4^\circ \]
Example The shaft shown in the figure is rotating with a speed of 10 rad/sec. Calculate the amount and angular orientation of the balance masses to be placed at a radial distance of 0.2 m.

\[ \sum \vec{M} = 0; \quad \vec{M}_1 + \vec{M}_2 + \vec{M}_{RCP} = 0 \]

\[ M_1 = F_1 l_1 = 40 \times 0.25 = 10 \text{ Nm} \angle 270^\circ \]

\[ M_2 = F_2 l_2 = 40 \times 0.5 = 20 \text{ Nm} \angle 90^\circ \]

\[ M_{RCP} = ? \]

\[ M_{RCP} = 22.5 \text{ Nm} \angle 296.5^\circ \]

\[ F_{RCP} = \frac{22.5}{0.75} = 30 \text{ N} \angle 296.5^\circ \]

\[ F_{RCP} = m_{LCP} \times r_{LCP} \times \omega^2 \implies m_{LCP} = \frac{30}{100 \times 0.2} = 1.5 \text{ kg} \angle 296.5^\circ \]

\[ \sum \vec{F} = 0; \quad \vec{F}_1 + \vec{F}_2 + \vec{F}_{RCP} + \vec{F}_{LCP} = 0 \]

\[ F_1 = m_1 r_1 \omega^2 = 4 \times 0.1 \times 10^2 = 40 \text{ N} \angle 270^\circ \]

\[ F_2 = m_2 r_2 \omega^2 = 2 \times 0.2 \times 10^2 = 40 \text{ N} \angle 90^\circ \]

\[ F_{LCP} = ? \]

\[ F_{LCP} = 30 \text{ N} \angle 33^\circ \]

\[ F_{LCP} = 30 = m_{LCP} \times r_{LCP} \times \omega^2 \implies m_{LCP} = \frac{30}{100 \times 0.2} = 1.5 \text{ kg} \angle 334^\circ \]
Example Balance the shaft shown in the figure dynamically by putting balance masses at a radial distance of 0.1 m. \( m_1 = 2 \text{ kg}, \ m_2 = 1 \text{ kg}, \ r_1 = 0.1 \text{ m}, \ r_2 = 0.2 \text{ m}. \)

\[
\sum M_{\text{LCP}} = 0; \quad \sum M_{\text{RCP}} + M_1 + M_2 = 0
\]

where

\[
M_1 = l_1 F_1 = 0.5 \times 0.2 = 0.1 \text{kgm}^2 \uparrow \\
M_2 = l_2 F_2 = 1.0 \times 0.2 = 0.2 \text{kgm}^2 \downarrow \\
M_{\text{RCP}} = l_{\text{RCP}} F_{\text{RCP}} = ?
\]

\[
M_{\text{RCP}} = 0.1 \text{kgm}^2 = l_{\text{RCP}} F_{\text{RCP}} = 1.5 \times F_{\text{RCP}}
\]

\[
F_{\text{RCP}} = \frac{0.1}{1.5} = 0.0667 \text{ kgm} \uparrow \\
m_{\text{RCP}} = \frac{F_{\text{RCP}}}{r_{\text{RCP}}} = \frac{0.0667}{0.1} = 0.667 \text{ kg} \uparrow \\
\]

\[
\sum \mathbf{F} = 0; \quad \sum \mathbf{F}_{\text{LCP}} + \mathbf{F}_{\text{RCP}} + \mathbf{F}_1 + \mathbf{F}_2 = 0
\]

where

\[
F_1 = m_1 r_1 = 2 \times 0.1 = 0.2 \text{kgm} \uparrow \\
F_2 = m_2 r_2 = 1 \times 0.2 = 0.2 \text{kgm} \downarrow \\
F_{\text{RCP}} = 0.0667 \text{ kgm} \uparrow \\
F_{\text{LCP}} = m_{\text{LCP}} r_{\text{LCP}} = ?
\]

\[
f_{\text{LCP}} = \frac{F_{\text{LCP}}}{r_{\text{LCP}}} = \frac{0.0667}{0.1} = 0.667 \text{ kg} \downarrow \\
\]

ANS
4- The shaft shown in the figure is rotating with a speed of 10 rad/sec. Calculate the amount and angular orientation of the balance masses to be placed at a radial distance of 0.2 m.

\[ m_1 = 2 \text{ kg}, m_2 = 2 \text{ kg}, m_3 = 2 \text{ kg}, r_1 = 0.2 \text{ m}, r_2 = 0.2 \text{ m}, r_3 = 0.2 \text{ m} \]

\[
\sum \vec{F} = 0: \quad F_{LCP} + F_{RCP} + F_1 + F_2 + F_3 = 0
\]

where

\[ F_1 = m_1 r_1 \omega^2 = 2 \times 0.2 \times 10^2 = 40 \text{ N} \leftarrow \]
\[ F_2 = m_2 r_2 \omega^2 = 2 \times 0.2 \times 10^2 = 40 \text{ N} \rightarrow \]
\[ F_3 = m_3 r_3 \omega^2 = 2 \times 0.2 \times 10^2 = 40 \text{ N} \downarrow \]
\[ F_{LCP} = m_{LCP} r_{LCP} \omega^2 = ? \]
\[ F_{RCP} = m_{RCP} r_{RCP} \omega^2 = ? \]

\[ m_{LCP} = \frac{F_{LCP}}{r_{LCP} \omega^2} = \frac{16.2}{0.2 \times 10^2} = 0.81 \text{ N} \angle 44.6^\circ \]

\[ m_{RCP} = \frac{F_{RCP}}{r_{RCP} \omega^2} = \frac{30.86}{0.2 \times 100} = 1.54 \text{ kg} \angle 112^\circ \]

\[
\sum \vec{M} = 0: \quad M_{RCP} + \vec{M}_1 + \vec{M}_2 + \vec{M}_3 = 0
\]

where

\[ M_1 = l_1 F_1 = 0.4 \times 40 = 16 \text{ Nm} \leftarrow \]
\[ M_2 = l_2 F_2 = 0.8 \times 40 = 32 \text{ Nm} \rightarrow \]
\[ M_3 = l_3 F_3 = 1.0 \times 40 = 40 \text{ Nm} \downarrow \]
\[ M_{RCP} = I_{RCP} F_{RCP} = ? \]
\[ M_{RCP} = 43.2 \text{ N} = I_{RCP} F_{RCP} = 1.4 \times F_{RCP} \]
\[ F_{RCP} = \frac{43.2}{1.4} = 30.86 \text{ N} \angle 112^\circ \]
\[ m_{RCP} = \frac{F_{RCP}}{r_{RCP} \omega^2} = \frac{30.86}{0.2 \times 100} = 1.54 \text{ kg} \angle 112^\circ \]